Localized Construction of Connected Dominating Set in Wireless Networks

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Abstract

Due to the fact that there is no fixed infrastructure or centralized management in ad hoc wireless networks, a Connected Dominating Set (CDS) of the graph representing the network is widely used as the *virtual backbone* of the network. Constructing a minimum CDS is NP-hard. In this paper, we propose a completely localized distributed algorithm which is r-CDS to construct a CDS with constant performance ratio. Our algorithm is better at maintenance since there is no need to build a tree or select a leader which are the common methods used in most of the distributed but serialized algorithms. We also compare our algorithm with other localized algorithms. The theoretical and simulation results show that our algorithm has a better performance ratio and constructs a CDS with smaller size in most cases.

1 Introduction

There is no fixed infrastructure or centralized management in wireless networks and the hosts may turn on, turn off or move around at any speed freely. Thus the network topology changes dynamically, i.e. the routing protocols in such a network need to adapt quickly to the topology changes. A host u can directly transmit messages to its neighbors within the transmission range. In this case, there is a link from u to every neighbor within the transmission range. If two hosts in the network are not within the transmission range of each other and they want to communicate, a multi-hop routing mechanism is required where some other hosts will relay the messages to the destination. All of these characteristics stimulate the use of the Connected Dominating Set (CDS) as a virtual backbone in the network.

We use $G = (V, E)$ to represent an ad hoc wireless network. V is the set of mobile hosts in the network and E represents all the links in the network. We assume that all the hosts are deployed in a 2-D plane and their maximum transmission range are the same. Thus the resultant topology graph of the network is modelled as an undirected *Unit Disk Graph* (UDG) [5]. In the context of graph theory, we call a host as a node. A Dominating Set (DS) of a graph $G = (V, E)$ is a subset $S \subset V$ such that each node either belongs to S or is adjacent to at least one node in S. A CDS is a DS which induces a connected subgraph. The nodes in the CDS are called the *dominators*, otherwise, *dominatees*. It is desirable to build a Minimum-sized Connected Dominating Set (MCDS). The construction of an MCDS in a UDG is proved to be NP-hard in [5]. Figure 1 gives an example UDG containing a CDS which is also a MCDS.

With the help of the CDS, the routing is easier and can adapt quickly to the topology changes. Only the nodes in the CDS need to maintain the routing information. Furthermore, if there is no topology changes in the subgraph induced by the CDS, there is no need to update the routing information. If a dominatee wants to deliver a message to another dominatee, it first sends the message to its dominator. Then the search space for the route is reduced only within the CDS. After the message is relayed to the destination's dominator, this dominator will deliver the message to the destination.

To construct a CDS, we utilize an Maximal Independent Set (MIS) which is also a subset of all the nodes in the network. The nodes in an MIS are pairwise nonadjacent and no more nodes can be added to remain the non-adjacency property. Thus each node

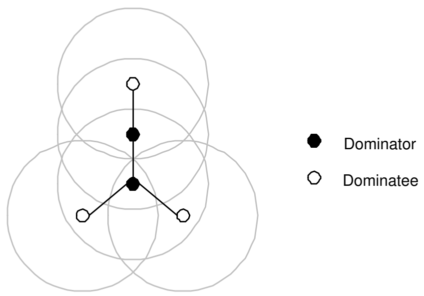

Figure 1: A UDG with a CDS

which is not in the MIS is adjacent to at least one node in the MIS. Thus an MIS is a DS. If we connect the nodes in an MIS, it forms a CDS.

The remainder of this paper is organized as follows. Section 2 briefly describes the related research works on the CDS problem. The r-CDS and the analysis of the algorithm are illustrated in Section 3. The simulation results are presented in Section 4. Section 5 ends this paper with conclusion and some future research work.

2 Related Work

The idea of using a CDS as a virtual backbone for routing is proposed in [8]. In most of the CDS construction algorithms, a coloring mechanism is used where initially all the nodes are white, a dominator is colored black and a dominatee is colored grey.

Guha and Khuller [11] first propose two centralized greedy algorithms. The number of white neighbors of each node or a pair of nodes (a dominatee with one of its white neighbor) is the greedy function. The one with the largest such number will become dominator(s) at each step. In the first algorithm, the CDS is built up at one node, then the searching space for the next dominator(s) is restricted to the current dominatees and the CDS expands until there is no white nodes. In the second algorithm, all the possible dominators are determined in the first phase, then they are connected through some intermediate nodes in the second phase. Both of these algorithms require the global information which is not practical for wireless networks. Das *et al.* [6, 7, 14] then give the implementations of the algorithms in [11]. They also mention the maintenance of the CDS if nodes have mobility.

Alzoubi *et. al.* [1, 2, 15] make a great improvement by using an MIS to design some CDS construction algorithms. They are two 2-phase distributed but serialized algorithms. They first construct a spanning tree and then iteratively label each node in the tree as either a dominator or a dominatee. They apply the algorithms in a UDG to obtain a constant performance ratio which is 8. Here the property that a node in a UDG is adjacent to at most 5 independent neighbors [12] is used to prove the upper bound of the performance ratio. Since a tree-structure is employed in these algorithms, this makes the maintenance of the CDS difficult. For example, hosts can turn off and on at any time and hosts may move around. Once these happen, the tree crashes and a new tree needs to be constructed.

Cardei *et. al.* [4] present a 2-phase distributed algorithm. This algorithm requires a leader to be selected at the beginning of the first phase. The leader first becomes a dominator making its neighbors dominatees. They introduce a new *active* state for white nodes. A white node becomes activate only after one of its neighbors becomes a dominatee. All the active nodes will compete to become a dominator based on the pair (the number of white neighbors, ID). The improvement over Alzoubi *et. al.*'s algorithms is that the root do not need to wait for the COMPLETE messages from the furthest nodes. The root initiates the connecting phase just after it receives NUMOFBLACKNEIGHBORS from all of its neighbors. A Steiner tree is used to connect the black nodes generated in the first phase. This process begins from the leader to the furthest node of the leader. Thus this algorithm is not a real localized algorithm, but a serialized one. They also apply the algorithm in a UGD and use the property [12] to obtain the performance ratio.

Alzoubi *et. al.* [3] also notice the difficulty of the maintenance of the previous algorithms. They then design a real localized 2-phase algorithm which is good at maintenance. An MIS is generated in a distributed fashion without building a tree or selecting a leader. Once a node knows that it has the smallest ID within its 1-hop neighborhood, this node becomes a dominator. After there are no white nodes, the dominators are responsible for identifying a path to connect all the dominators. In this algorithm, no network connectivity information is utilized.

Wu and Li [16] also propose a completely localized algorithm where each node knows the connectivity information within the 2-hop neighborhood. If a node has two unconnected neighbors, it becomes a dominator. The generated CDS is easy to maintain. But the size of the CDS is large. Thus they give two rules to prune the generated CDS. The performance ratio is not specified. In [15], the authors give the performance ratio of this algorithm and correct the time complexity and message complexity.

Table 1 lists some measurement parameters of these algorithms where Δ is the maximum degree in the graph; $|C|$ is the size of the resultant CDS; n and m are the number of the vertices and edges in the graph and opt is the size of an optimal MCDS. PR stands for the performance ratio.

	PR	Time	Message	Maintenance
[2]-I	$12opt + 3$	$O(n)$	$O(n\log(n))$	Non-localized
[2]-II	$8opt + 1$	$O(n)$	$O(n\log(n))$	Non-localized
[4]	$8opt$	$O(n)$	$O(n\Delta)$	Non-localized
[3]	$192opt + 48$	$O(n)$	$O(n)$	Localized
[16]	$O(n)$	$O(\Delta^3)$	$\Theta(m)$	Localized

Table 1: measurement parameters of the CDS construction algorithms in [2, 4, 3, 16].

In this paper, we propose a completely localized 1-phase algorithm which is called r-CDS. Compared with the serialized algorithms, our algorithm is better at maintenance since there is no need to build a tree or select a leader. The simulation results show that the size of the CDS constructed using our algorithm is comparable to that by using the serialized algorithms. Compared with the localized algorithms, our algorithm also gives a smaller CDS and a smaller constant performance ratio.

3 Algorithm Description

r-CDS is a localized one-phase algorithm where each node only needs to know the connectivity information within its 2-hop-away neighborhood. Initially, all the nodes are white. We classify the dominators into two sub-classes which are *dominator1 nodes*, and *dominator2 nodes* that connect all the dominator1 nodes. The dominator1 and dominator2 nodes are colored black, and the *dominatees* are colored grey. All the dominator1 nodes form an MIS and all the dominators form a CDS.

We use the following terms in the r-CDS algorithm,
$d(u)$ = The degree of node u.
$deg(u)$ = The number of the white neighbors of node u which is also called the *effective degree* of node u.
$r(u)$ = The number of 2-hop-away neighbors - $d(u)$. (r for brevity)
$N(u)$ = All of node u's 1-hop-away neighbors excluding node u which is also called the open neighbor set of node u.
$N[u] = N(u) \cup \{u\}$ which is also called the close neighbor set of node u.

We assume that each node knows its 1-hop-away neighborhood connectivity information. This information can be periodically collected through *hello* messages. Every node will exchange this information with all of its 1-hop-away neighbors so that r can be computed at each node. Then the r-CDS is applied at each node as elaborated below. Eventually, each node will become a dominator1, a dominator2 or a dominatee node.

Initially, all the nodes are white. Each node u first broadcasts its r. After receiving all the 1-hop-away neighbors' r's, u can compare its r with its white neighbors' r's. If node u has the smallest r, u becomes a dominator1 node and broadcasts a BLACK message. If there exists any other nodes whose r is equal to $r(u)$, u will become a dominator1 node if u has the largest $deg(u)$. If there still exist ties, u will become a dominator1 node if u has the smallest ID. Then u broadcasts a BLACK message. So the node with the smallest tuple (r, deg, ID) become a dominator1 node.

Upon receiving a BLACK message from its neighbor v, a white node u is colored grey and u broadcasts a GREY message containing the pair (u's ID, v's ID).

Upon receiving a BLACK message from a node w, if a grey node u has already received a notification that there is a 2-hop-away black neighbor v sent by a neighbor x and v has not been connected to w yet, then u and x become dominator2 nodes to connect node v and node w.

Upon receiving a GREY message or a BLACK message, a white node u decrements its effective degree $deg(u)$ by 1 and broadcasts $deg(u)$.

If a black node u receives a GREY message and a black node w's ID from a grey node v and if w has not been connected to u, v becomes a dominator2 node to connect u and w.

Note that the dominator1 nodes and the dominator2 nodes are decided in the same phase, unlike the algorithms in [15] and [4] where two phases are needed to decide the dominating nodes and to connect the dominating nodes using connectors. No tree-structure is needed, no nodes need to wait and each node can conduct this procedure at the same time.

We will illustrate the correctness, the message and time complexities and the performance ratio through the following lemmas and theories.

Lemma 3.1 *All the dominator1 nodes form an MIS.*

Proof: In the domination decision procedure, only the white node with the smallest (r, deg, id) will become a dominator1 node. Therefore, no two white nodes become dominator1 nodes at the same time. In addition, if a node u is black, $\forall v \in N(u), v$ is grey. Thus, any pair of dominator1 nodes are disjointed and form an Independent Set. Each dominatee and dominator2 node is adjacent to at least one dominator1 node. So none of them can become a dominator1 node. Thus all the dominator1 nodes form an MIS. ■

Lemma 3.2 *Let d denotes the number of hops between any pair of dominator1 nodes, then $2 \leq d \leq 3$.*

Proof: Let D_1 be the set of all the dominator1 nodes. By Lemma 3.1, D_1 is an MIS. For $\forall u, v \in D_1, (u, v) \notin E$, thus $2 \leq d$. $\forall u \in V - D_1, u$ must have a neighbor $v \in D_1$, thus $d \leq 3$. ■

Theorem 3.3 *All the dominators form a CDS.*

Proof: An MIS is a DS. From Lemma 3.1, all the dominator1 nodes form a DS. In our algorithm, any pair of dominator1 nodes that are 2 or 3-hop away will be connected through a dominator2 node. By Lemma 3.2, all the domintor2 nodes connect all the dominator1 nodes. Thus all the dominators form a CDS. ■

Theorem 3.4 *The time complexity is $O(\Delta)$ where Δ is the maximum degree and the message complexity is $O(n\Delta^2)$.*

Proof: The main part that decides the time complexity is the one that checks whether two dominator1 nodes have been connected. This depends on the implementation. If each dominator1 node waits until gather all the information about the possible paths and then notify the node(s) on the shortest path to become dominator2 node(s), the time complexity is $O(n)$. In our implementation, a dominator1 node will notify the first node that indicates there exists another dominator1 node to be connected to become a dominator2 node. Although the path between a pair of dominator1 nodes may not be the shortest, the time complexity is improved to $O(\Delta)$. In the worst case, a node needs to wait $O(\frac{n}{2})$ time to color itself where all the nodes form a chain and their IDs are in descending or ascending order, as shown in Figure 2 (a). At the beginning, node 1 and node 8 become dominator1 nodes, then node 3 and node 5 become dominator1 nodes. All the nodes between a pair of dominator1 nodes become dominator2 nodes to form a CDS (Figure 2(b)). So the middle node in a chain may wait up to $O(\frac{n}{2})$ time to decide the color of itself.

When a grey node queries a black node whether another black node has been connected to the first black node, it may send $O(\Delta^2)$ messages. Thus the message complexity is $O(n\Delta^2)$ ■

The density D_n of packing n equal circles with diameter d in another large circle Q is the ratio of the total area of the circles to the area of Q denoted by $A(Q)$ and it is given in [9]:

$$D_n = \frac{nd^2\pi}{4A(Q)} \tag{1}$$

Let D be the maximum density of packing n equal circles in a circle. The upper bound of D is given by Molnar in [13]:

$$D \leq \frac{n}{\left[1 - \frac{\sqrt{3}}{2} + \sqrt{\frac{3}{4} + \frac{2\sqrt{3}}{\pi}(n-1)}\right]^2} \tag{2}$$

Lemma 3.5 *Let S be any MIS of a UDG G. For any node $u \in S$, the number of the nodes in S that are at most three hops away from u is at most 42.*

Proof: To decide the number of nodes that are at most three hops away can be transformed to the packing problem shown in Figure 3. Let m be the number

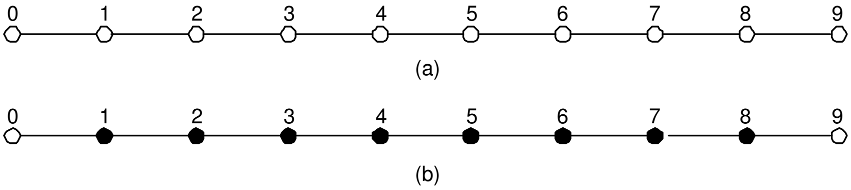

Figure 2: A worst case example

of the nodes in S that are at most three hops away from u, r be the transmission range of each node, *i.e.*, the radius of each small circle where $r = 0.5$. All the nodes that are at most three hops away from u must lie in the circle C with radius $R = 3.5$ except the small circle with radius $R = 0.5$. By equation 1 and 2, $m \leq 43 - 1 = 42$. ■

Theorem 3.6 *The performance ratio of our algorithm is 172.*

Proof: The size of any MIS S in a graph is at most $4opt + 1$ where *opt* is the size of any optimal CDS of the graph [15]. Let C be the CDS obtained by our algorithm. By Lemma 3.5, $|C| \leq 2 \cdot \frac{42|S|}{2} + |S| \leq 172opt + 43$. ■

We extend the rules in [16] to prune the obtained CDS. Each dominating node can conduct these rules after becoming a dominator2 node, or when a dominator node receives a dominator2 message. So there is no need to add another phase to prune the CDS.

RULE1: A node u can be changed to a dominatee and colored grey if there exists a black node v such that either $N[u] \subset N[v]$, or $N[u] = N[v]$ and u has the minimum ID.

RULE2: A node u can be changed to a dominatee and colored grey if u has 2 black neighbors v and w such that either $N(u) \subset N(v) \cup N(w) - \{u\}$, or $N(u) - \{v, w\} = N(v) \cup N(w) - \{u, v, w\}$ and u has the minimum ID.

RULE3: A node u can be changed to a dominatee and colored grey if u has 3 black neighbors x, y and z such that either $N(u) \subset N(x) \cup N(y) \cup N(z) - \{u\}$, or $N(u) - \{x, y, z\} = N(x) \cup N(y) \cup N(z) - \{u, x, y, z\}$ and u has the minimum ID.

In Figure 4, node 1, node 2, and node 3 are dominators. If we apply RULE2 in [16] to this example, no dominator can be changed to a dominatee. We fix this problem by checking the IDs of the compared dominators only when their open neighbor set are the same. In this example, either node 2 or node 3 can be changed to a dominatee by applying RULE2.

Theorem 3.7 *The prune phase does not break the properties of the CDS obtained by applying the domination decision procedure.*

Proof: For RULE1, since $N[u] \subseteq N[v]$, v is adjacent to all u's neighbors and u. Thus, N[u] can all be dominated by v. In addition, if u is a dominator2 node between v and w where $w \in N(u)$, then v is adjacent to w as well. Therefore, the CDS properties are still preserved when u becomes a dominatee. For RULE2, since $N(u) \subseteq N(v) \cup N(w)$, v and w can dominate all nodes in $N(u)$. Since v and w are u's neighbors, u can be dominated by either v or w. Thus, the DS property is preserved. In addition, v is adjacent to w. Hence, it does not break the connectivity property either. For RULE3, since $N(u) \subseteq N(x) \cup N(y) \cup N(z)$, there must exist a path between any pair of the nodes in $\{x, y, z\}$. Thus, the connectivity of x, y, z and the rest of the dominators will not be broken when u becomes a dominatee. ■

Theorem 3.8 *To get the best possible pruned result, Rule2 in [16] cannot be extended to more than 3 neighbors.*

Proof: A counterexample is given in Figure 5 to show that the connectivity will not be preserved if the rule is extended to more than 3 neighbors. ■

Figure 6 gives an example of constructing a CDS using this algorithm. Initially, all the nodes are white. Node 0 and node 3 become dominator1 nodes and broadcast a BLACK message since they have the smallest r's and IDs within the local neighborhood (Figure 6 (a)). After receiving a BLACK message, node 1, node 2, node5 and node 6 become dominatees and broadcast node 0's ID with a GREY message. Node 4 also becomes a dominatee and broadcasts node 3's ID with a GREY message. Suppose node 2 first notice the fact that node 0 and node 3 are

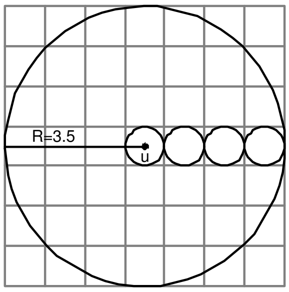

Figure 3: Packing circles with $r = 0.5$ in a circle with $R = 3.5$

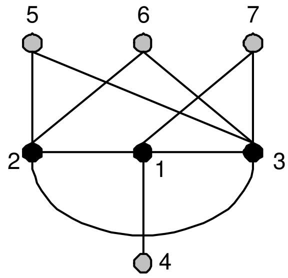

Figure 4: An example for pruning the CDS

two unconnected black nodes. Then node 2 and node 5 become dominator2 nodes (Figure 6 (b)). Node 2 and node 3 change back to dominatees according to RULE1 (Figure 6 (c)). Finally, node 0 and node 5 form the CDS. Scheduling of the message transmissions may affect the resultant CDS. For example, if node 5 first notice the fact that node 0 and node 3 are not connected, node 5 will become a dominator2 node without making node 2 a dominator2 node. But after applying RUE1 the resultant CDS is the same as the previous one.

4 Simulation Results

The size of the generated CDS is the criteria to measure a CDS generation algorithm. In this simulation, we compare the size of the CDS obtained by r-CDS with the ones in [3] (denoted as A-CDS) and [16] (denoted as W-CDS) since these algorithms are localized ones.

Totally N hosts are randomly generated in a fixed 100*100 2-D square. The transmission range of each hose is R. The generated CDS is C and $\mid C \mid$ is the size of the CDS. Only the connected networks are considered in this simulation. The algorithms are run 100 times for each group of N and R and the results averaged. We also evaluate the effect of the 3 rules in this simulation.

First, for fixed R, we compare the sizes of C's generated by these three algorithms with different N's. In this simulation, the three Rules are not applied for r-CDS. The result for $R = 25$ is shown in Figure 7. In average, the mean $\mid C \mid$ of r-CDS is 41% lower than W-CDS's and 15% lower than A-CDS's. The improvement increases as the size of the network increases. When R is fixed, as N increases, the network generated expands a larger area, so more nodes are selected as the dominating nodes.

Then, for fixed N, we compare the sizes of C's generated by these three algorithms with different R's. In this simulation, the three Rules are not applied for r-CDS. The result for $N = 20$ is shown in Figure 8. In average, the mean $\mid C \mid$ of r-CDS is 66% lower than W-CDS's and 36% lower than A-CDS's. For $N = 50$, the mean $\mid C \mid$ of r-CDS is 79% lower than W-CDS's and 35% lower than A-CDS's. For $N = 75$, the mean $\mid C \mid$ of r-CDS is 83% lower than W-CDS's and 33% lower than A-CDS's. When N is fixed, as

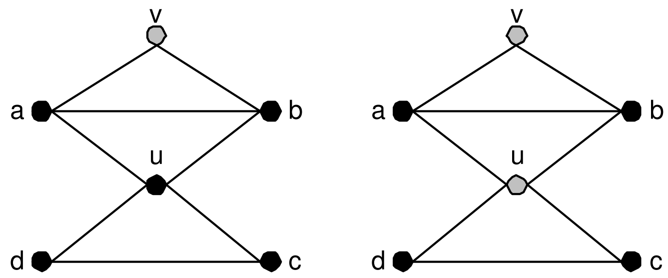

Figure 5: An counterexample

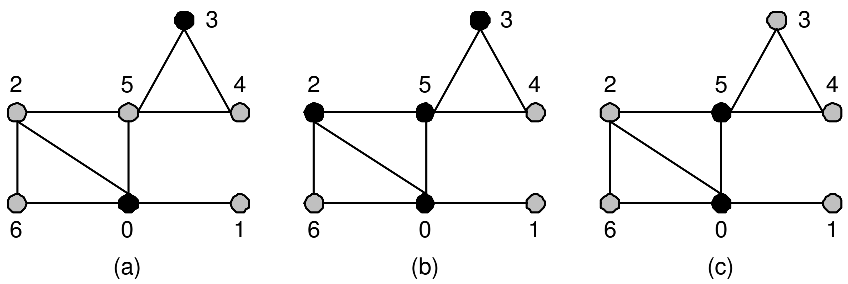

Figure 6: An example for CDS construction

R increases, the transmission range of each node can cover more neighbors, so less nodes are selected as the dominating nodes.

To evaluate the effect of the three Rules, we compare the sizes of C's with and without applying the Rules for different N's with fixed R and for different R's with fixed N. The results are shown in Figure 9 and Figure 10. For $R = 25$, in average, if applying Rule1, $\mid C \mid$ is 26% lower than the one not applying any Rule; if applying Rule1 and Rule2, $\mid C \mid$ is 37% lower and if applying all the Rules, $\mid C \mid$ is 40% lower. For $N = 60$, in average, if applying Rule1, $\mid C \mid$ is 25% lower than the one not applying any Rule; if applying Rule1 and Rule2, $\mid C \mid$ is 53% lower and if applying all the Rules, $\mid C \mid$ is 56% lower.

We also compare the the results for our algorithm and the ones in [2] and [4] given in Table 2. The results of [2] and [4] are from [4]. Since in [2] and [4], either a tree is built or a leader is selected initially, this makes the maintenance difficult.

From the simulation results, we can conclude that r-CDS generates a smaller CDS because r is defined based on the connectivity information of the network and ID of each host which is random and sometimes not an important parameter of a network are used only to break ties. To become a dominator, a node needs to have a small r, which means this node dominates a large number of nodes and the number of nodes not dominated by this node is small.

N	R	[2]	[4]	r-CDS
20	25	11.9	9.59	9.75
20	50	5.54	3.61	3.6
50	25	17.94	13.16	16.29
50	50	6.34	4.11	4.31
100	25	20.44	13.47	19.29
100	50	6.68	4.55	4.7

Table 2: Comparison of the results for our algorithm and the ones in [2] and [4].

5 Conclusion and Future Research Work

In this paper, we study the problem of constructing a CDS in wireless networks and propose a localized one-phase distributed algorithm which is r-CDS. The

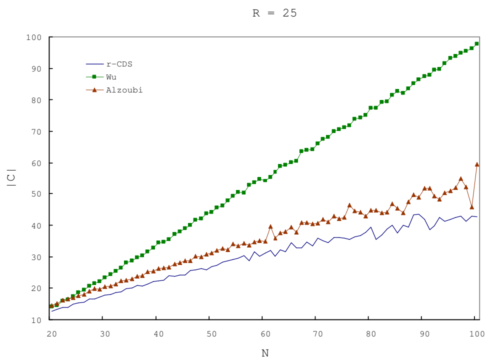

Figure 7: Simulation results for R=25

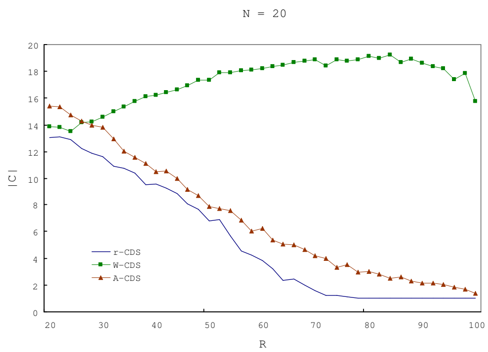

Figure 8: Simulation results for N=20

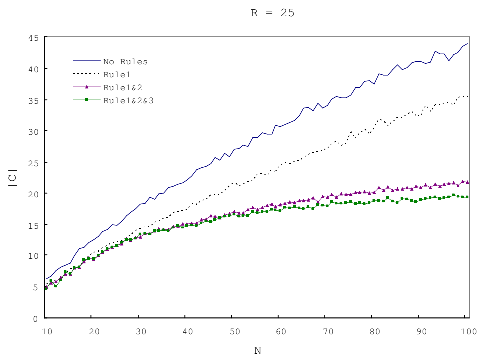

Figure 9: Simulation results for evaluating the three Rules when R=25

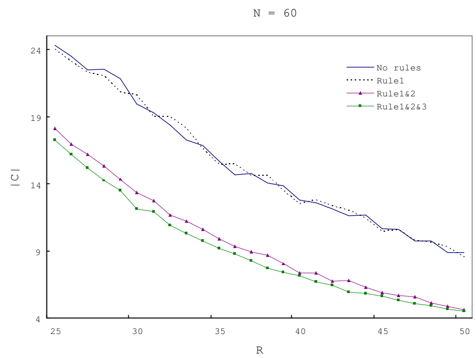

Figure 10: Simulation results for evaluating the three Rules when N=60

simulation results show that r-CDS outperforms the algorithms in [3] and [16] by generating a smaller CDS with a smaller constant performance ratio. It is our interest to further investigate the maintenance of the CDS and the routing based on the generated CDS. The work can also be extended to develop CDS construction algorithms when hosts in a network have different transmission ranges.

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